Clairaut's equation extended to fast rotators

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The determination of the internal structure of rotating self-gravitating bodies is a longstanding challenge in astrophysics (e.g. planets, stars, asteroids, even galaxies). As it is well known, the major difficulty is the calculation of the gravitational potential, which is analytically known for in a few cases only. In this context, homogeneous ellipsoids and spheroids in rigid rotation look particularly attractive as they correspond exactly to figures of equilibrium. In rocky/icy bodies like asteroids or dwarf/telluric planets, the mass density is expected to vary weakly, from the center to the surface (jumps may be present due to changes in the composition). In contrast, gaseous systems like giant planets and stars have large variations in their mass density profile. Classical theories mostly rely on the approximation of slow rotation.

In this talk, we will present the theory of Nested Spheroidal Figures of Equilibrium (Huré 2022a,b), which determines the conditions of equilibrium for heterogeneous systems made of $\mathcal{L}$ homogeneous spheroidal layers. We will focus on the limit case where the number of layer is infinite, which enables to model full heterogeneity. Provided isopycnic surfaces are close to confocal spheroids, we can derive an integro-differential equation linking the local flattening to the mass density profile. With this approach (validated numerically by the Self-Consistent Field method), fast rotators are easily reached. Clairaut's equation is recovered at small ellipticities, but we will show that it starts to fail when the axis ratio is below 0.95, typically (objects like Jupiter or Ceres are therefore concerned).