Topological modes in stellar oscillations

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Abstract

Certain peculiar waves in inhomogeneous fluids have been discovered using topological analysis. Recently, a Lamb-like wave has been theorized to exist in stratified compressible fluids, however it is hardly expected to propagate on Earth, neither in the atmosphere nor in oceans.

Q: Do Lamb-like waves propagate in stellar objects ? What methods are needed to understand them ?

Linear waves

We adapt tools that have been originally developed by the **topological insulator community** to study the seminal case of a **simple oscillating star** (adiabatic perturbations of a non-rotating, non-magnetic, stably stratified).

Perturbations evolution as a Schrödinger-like equation:

$$i\partial_t \boldsymbol{X} = \mathcal{H}\boldsymbol{X} \text{ with } \mathcal{H} = \begin{pmatrix} 0 & 0 & 0 & L_\ell(r) \\ 0 & 0 & iN & -iS + ic_\mathrm{s}\partial_r \\ 0 & -iN & 0 & 0 \\ L_\ell(r) & iS + ic_\mathrm{s}\partial_r & 0 & 0 \end{pmatrix}$$

$$N^2 \equiv -g\frac{\mathrm{dln}\,\rho_0}{\mathrm{d}r} - \frac{g^2}{c_\mathrm{s}^2} \qquad S \equiv \frac{c_\mathrm{s}}{2g}\left(N^2 - \frac{g^2}{c_\mathrm{s}^2}\right) - \frac{1}{2}\frac{\mathrm{d}c_\mathrm{s}}{\mathrm{d}r} + \frac{c_\mathrm{s}}{r}.$$

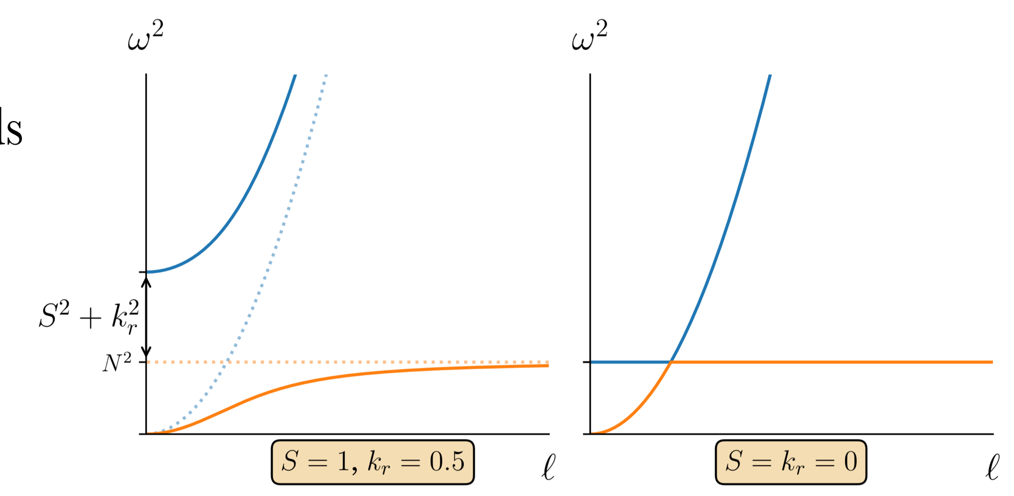
Local problem: Wigner transform

How to set up a local version of the problem ? Transpose it to a phase space, by **Wigner Transform**. It transforms the differential operator into a simple matrix, defined on a **phase space** $\{r, k_r\}$.

For this problem, it reads $H \equiv \begin{pmatrix} 0 & 0 & 0 & L_\ell \\ 0 & 0 & iN & k_r - iS \\ 0 & -iN & 0 & 0 \\ L_\ell & k_r + iS & 0 & 0 \end{pmatrix}$

The frequencies form two bands and a gap (left).

This gap closes for $S = k_r = 0$ (right).



Topology: Berry-Chern monopole

The Wigner matrix is also used to define a central quantity in topological physics: the **Berry curvature**. It is a real vector field, which reads for one of the eigenvectors

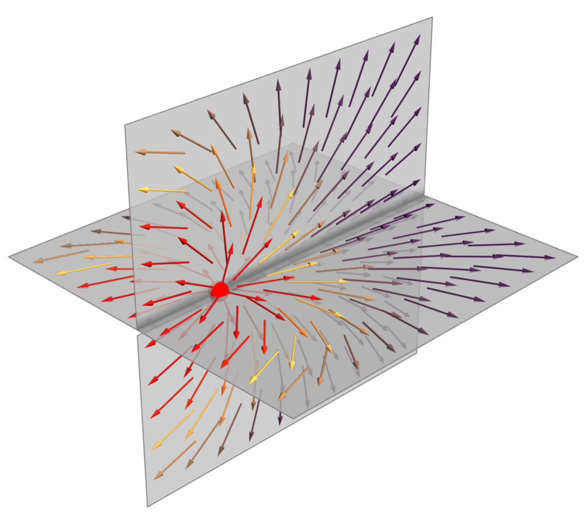
$$\boldsymbol{F} \equiv i\boldsymbol{\nabla} \wedge \left(\boldsymbol{X}^\dagger \cdot \boldsymbol{\nabla}\boldsymbol{X}\right)$$

(measurement of twisting of eigenvectors when parameters vary.)

There is a **topological obstruction** of defining the eigenvector's phase globally when the curvature is infinite.

The Berry curvature has a pole at the point:

$$S = k_r = 0, \text{ and } L_\ell = N$$



Associated **topological invariant**

$$\mathcal{C} = \frac{1}{2\pi}\oint_\Sigma \mathrm{d}\boldsymbol{\Sigma} \cdot \boldsymbol{F} = 1$$

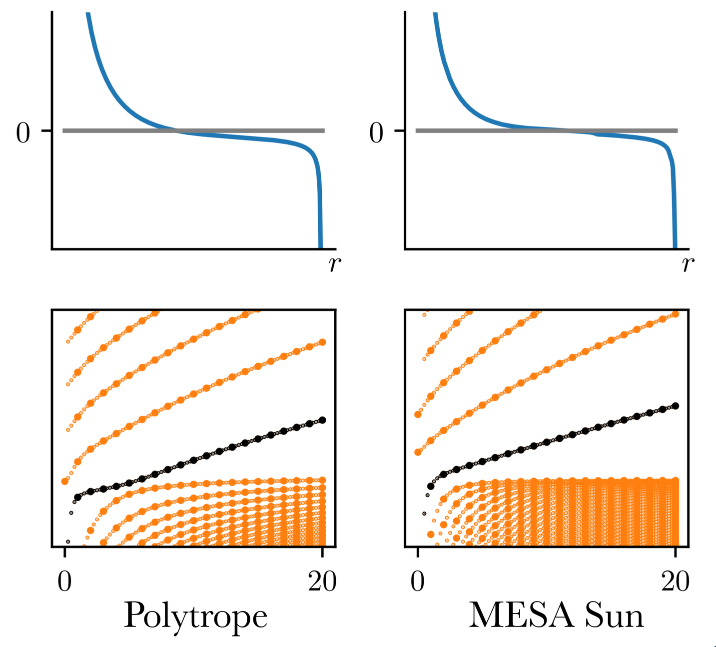
(flux of Berry curvature on a surface enclosing it)

Lamb-like wave

Topology theorem:
There are as many waves transiting between the bands, as $\mathcal{C}$.

Solving numerically for realistic models, we observe the propagation of 1 mode of topological origin: the Lamb-like wave.

Its existence is guaranteed for every object with **cancelling $S(r)$**, which is ubiquitous. (Geometrically and presence of edge)



A: The Lamb-like wave should propagate in every stellar object.
It is a bulk mode crossing the gap.
It is of topological origins.

Future prospects

- If it does propagate, the Lamb-like wave should have been simulated/detected already. Can it be found in simulations ?
- Hermiticity is crucial for topological machinery. What if the problem is not Hermitian ? (Effects of excitation and diffusion)
- Use of topology for instability studies: convection (connected to this problem), differential rotation (disks instabilities).

Publication

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