# Detecting targets with radial velocities for their atmospheric characterisation with the next flagship instruments

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# Key message

A core objective of flagship ESA and NASA mission at the 2050 horizon is to characterise the atmosphere of Earth-like exoplanets

Radial velocity measurements can pre-detect Earth-like targets and will be essential in any case to measure planetary mass

The radial velocity signal of the Earth on the Sun is 9 cm/s and current instruments are limited at  $\sim$  1 m/s because of complex noises

Reaching 9 cm/s precision is the primary goal of the radial velocity community, it is largely a data analysis problem, and there has been recent progress



# **Objectives**







16

H<sub>2</sub>O

20

Schwietermann+18

≥ 2

10

12

14

Wavelength  $[\mu m]$ 

-0.2 -0.1 time from mid-transit [days]

0.75 -

5.23 µm

0.0

0.1







### JWST, PCS@ELT LIFE (ESA), HWO (NASA),

...Non transiting

Characterizing the atmosphere of Earthanalogs from space: priority for the horizon 2050



# **Radial velocites are key**









- + Results obtained faster, more time for characterisation
- + Mitigates the risk of the missions
- + True for other kinds of planets





The yield of atmospheric characterization is greatly improved if Earth like planets are detected in advance, require targets  $\leq$  20 parsec



Mission time (days)

Morgan et al. 2021



5

# NASA Extreme precision radial velocity report Crass et al. 2021

Roadmap to the detection of Earth-like planets with radial velocities to prepare HWO

#### « Musts » = requirements

- Determine by 2025 the **feasibility** to detect Earth-mass planets in the habitable zone of solar-type stars
- Demonstrate by 2025 on sky precision of 30 cm/s
- Conduct precursor survey: now-2035 on 100 stars on the « green target list »

#### « Wants »

- Survey as many stars as possible (« Yellow list: 100 stars »)
- Least estimated cost

Green target list (106 stars)

- Spectral types F7–K9 and
- vsin(i) < 5 km/sullet
- Close (On the HabEx 'deep survey' or '50 highest priority stars' lists (Gaudi et al. 2020), or on at least 2 other mission concept target lists (including LUVOIR-A, LUVOIR-B, HabEx 'master list,' Starshade Rendezvous)



# NASA Extreme precision radial velocity report

Simulation with different telescope/instrument associations

In principle the 100 prioritised targets can all be characterised

Simulation done in the best case scenario: White noise



Histogram of obtained signal to noise ratio for 100 priority stars assuming they all have an Earth



Crass et al. 2021



# **Excerpts from the EPRV report**

Telluric line contamination is assumed to be limited and correctable

Weather conditions are uncorrelated.

We do not account for errors due to RV contamination from additional planets in the same planetary system

Stellar variability is assumed to be adequately characterized and mitigated during the data extraction of RV signals such that the remaining signal due to stellar variability is uncorrelated in time.

Extreme precision radial velocity is very useful if these problems can be mitigated



# Noise in radial velocities is not white





# Key ideas

### Planets induce a pure Doppler shift



### Planets induce a periodic signal

RV data and max. likelihood model





# periodic

Stellar and instrumental effects are (usually) not strictly



RV data and max. likelihood model









### **Modelling:** which noise properties, priors?





### Are planets detected? Which ones?



Hara & Ford, Annual Reviews of statistisics and Its Application: what are the problems to solve?





# An optimal exoplanet detection criterion

1000 radial velocity datasets with 0,1 or 2 planets

- Analysed with the same model
- With different detection criteria

### Bayes factors and FAPs

- **Optimal?**  $\bullet$ Which criterion maximises true detections?
- Do not encode where the planet is
- Are not defined on a very intuitive scale





# to detection of n > 0 components with parameters in $\Theta_i$ , i = 1...n, the Las n "bore". We must the valuate the utility of this clim if the true tests $f_{1...,0}$ , here $r_{1}$ gives effect to one provide onside from such to the relevant to valuate the utility of this clim if the true to claim twice the retection of a given component. Furthermore, if a

number of correct detections. Suppos components, one in  $\Theta_1$ , one in  $\Theta_2$ , with a non empty inter parameters are such that  $heta_1$  belongs to  $\Theta_1 \cap \Theta_2$  and  $heta_2$ whether we associate  $\theta_1$  to  $\Theta_1$  or  $\Theta_2$ , we have two or only case. we choose the injection which leads to as many corr

note by  $A_m^k$  the region of parameter space with k componen component in each of the  $\Theta_i$ , i = 1..m,  $m \leq n$ .

If k planets are truly present in the data, n detections are cmeans that the true detections of  $\min(k, n) - i$  are missed. W adding a term  $-\beta(\min(k, n) - i)$  whenever it happens. The e

$$\begin{split} E_{\theta,\eta} \left[ U\left\{ a, (\theta, \eta) \right\} \right] &= -n\alpha p(0 \mid y) \\ (20) \\ &+ \left[ -(n-1)\alpha I_{A_1^1} - (n\alpha + \beta) \left( 1 - I_{A_1^1} \right) \right] p(1 \mid y) \\ (21) \\ &+ \left[ -(n-2)\alpha I_{A_2^2} - ((n-1)\alpha + \beta) I_{A_1^2} - (n\alpha + 2\beta) \left( 1 - (n\alpha + 2\beta) \right) \right] \\ (22) \end{split}$$

.  
(23)  
+ 
$$\left[\sum_{i=1}^{k} (-(n-i)\alpha - (k-i)\beta)I_{A_i^k} - (n\alpha + k\beta)\left(1 - \sum_{i=1}^{k} (1 - \sum_{i$$

$$\begin{array}{c} \text{LEMMA} \\ \text{(25)} \\ + \left[\sum_{i=1}^{n} -(n-i)(\alpha+\beta)I_{A_{i}^{n}} - n(\alpha+\beta)\left(1-\sum_{i=1}^{n}I_{A_{i}^{n}}\right)\right] \\ \text{(26)} \\ + \left[\sum_{i=1}^{n} -(n-i)(\alpha+\beta)I_{A_{i}^{n+1}} - n(\alpha+\beta)\left(1-\sum_{i=1}^{n}I_{A_{i}^{n+1}}\right)\right] \\ \text{We begin} \\ \text{REMARK} \\ \text{sen between} \\ \text{the parametric} \\ \text{the parametric} \end{array}$$

 $+\left[\sum_{i=1}^{n} -(n-i)(\alpha+\beta)I_{A_{i}^{n}} - n(\alpha+\beta)\left(1 - \sum_{i=1}^{n}I_{A_{i}^{n}max}\right) - (n_{max} - n_{max})\right]$ We denote by m the maximum number of different  $\theta_i$ s the Re-arranging the terms, we have (29

) 
$$E_{\theta,\eta}[U\{a,(\theta,\eta)\}] = -n\alpha + (\alpha + \beta)\sum_{i=1}^{n} iI_{A_i} - \beta\sum_{k=1}^{n_{max}} k$$

$$= -(\alpha E[\text{FD}] + \beta E[\text{MD}])$$

where E[FD] and E[MD] are the expected numbers of false detections and 1 when claiming the detection of components with parameters in  $\Theta_1, ..., \Theta_n$ 

1) 
$$E[\text{FD}] = n - \sum_{i=1}^{n} iI_{A_i}$$
2) 
$$E[\text{MD}] = \bar{n} - \sum_{i=1}^{n} iI_{A_i},$$

where  $\bar{n} := \sum_{k=1}^{n_{max}} kp(k \mid y)$  does not depend on the number of componer suming that  $\alpha \neq 0$  (or equivalently  $\alpha > 0$ , since  $\alpha$  is non negative), we ca by  $\alpha$ . Denoting by  $\gamma = \beta / \alpha$ , without loss of generality we can maximize

(33) 
$$E_{\theta,\eta} \left[ U\left\{ a, (\theta, \eta) \right\} \right] = -n + (1+\gamma) \sum_{i=1}^{n} i I_{A_i} - \gamma \bar{n}.$$
Where  $\bar{n}$  does not depend on the number of planets

CONTINUOUS MULTIPLE HYPOTHESIS TESTING FOR

where 
$$[\![1,n]\!]_j$$
 is a draw of j indices without replaceme

(36) 
$$I_{A_j} = \sum_{k_1, \dots, k_j \in \llbracket 1, n \rrbracket_j} I_{\Theta_{k_1} \land \Theta_{k_2} \land \dots \land}$$

(7) 
$$\sum_{i=1}^{n} I_{\Theta_i} = \sum_{i=1}^{n} \sum_{j=0}^{n-1} \sum_{k_1, \dots, k_j \in [[1,n]]_j \setminus \{i\}} I_{\Theta_i \land i}$$

In this sum, the term  $I_{\Theta_1 \land \Theta_2 \land ... \land \Theta_n}$  appears n times, t pear n-1 times, so we obtain the desired result.

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 $\}$  where T

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PROOF. With the notation above, we have seen that 
$$u_n$$
 is increasing,  $v_n$  is decreasing,  
and  $v_{n-1} - v_n$  is decreasing. Furthermore, by hypothesis  $u_n - u_{n-1}$  is increasing, which by  
definition of  $v'_n$  means that  $v'_{n-1} - v'_n$  is decreasing. In the following, we reason on  $v_n$  but  
the argument is identical if  $v_n$  is replaced by  $v'_n$ .  
Let us for  $n \ge 0$ . The constrained method is

 $\min v_n$  subject to  $u_n <= x$ 

while the maximum utility problem can be

the argument is identical if  $v_n$  is replaced by  $v'_n$ .

Let us fix x > 0. The constrained problem is

$$\min_{n}$$

Since  $u_n$  is increasing, there is a highest  $n_0$ constraint, i.e. such that  $u_{n_0} \ll x$ . Since problem is found for  $n = n_0$ , for any con choose  $\gamma$  such that the solution of the maxi ration satisfying the constraint. We show th to a larger value of  $v_n + \frac{1}{2}u_n$ .

From our hypotheses, we see that the rat  $n \leq n_0$ , we will have

$$v_n + \frac{1}{\gamma}u_n$$

if

(47) 
$$\gamma \ge \frac{u}{v}$$

Note that if  $v_{n_0} - v_{n_0+1} = 0$ , since  $v_n - v_n$ . also  $v_n - v_{n+1} = 0$ , and we can always re such that  $v_n - v_{n+1} \neq 0$ . For  $n > n_0$ , we w

$$v_n + \frac{1}{\gamma}$$

These two conditions can be satisfied sin

$$\frac{u_{n_0} - u_{n_0-1}}{v_{n_0-1} - v_{n_0}}$$

 $\gamma \leq$ 

Choosing  $\gamma$  between those two bounds g increasing function of x.

As long as the sequence  $(u_{n+1}^y - u_n^y)_{n=1}$ the constrained problem have the same solu but can be ensured under the following con

LEMMA E.4. If 
$$\forall n > 0$$
,  $\exists i_0, \forall j = 1...r$ 

**PROOF.** Let us suppose that  $\exists n \ge 1$  such that  $u_{n+1} - u_n < u_n - u_{n-1}$ . Replacing by the explicit expression of  $u_n$ , the inequality is equivalent to

(48) 
$$\sum_{i=1}^{n} I_{\Theta_{i}^{n}} - \sum_{i=1}^{n+1} I_{\Theta_{i}^{n+1}} < \sum_{i=1}^{n-1} I_{\Theta_{i}^{n-1}} - \sum_{i=1}^{n} I_{\Theta_{i}^{n-1}}$$

By hypothesis,  $\exists i_0, \forall j = 1...n - 1, \Theta_{i_0} \cap \Theta_j^{n-1} = \emptyset$ , Eq. (48) can be written

9) 
$$\sum_{i=1}^{n} I_{\Theta_{i}^{n}} - \sum_{i=1, i \neq i_{0}}^{n+1} I_{\Theta_{i}^{n+1}} < I_{\Theta_{i_{0}}^{n+1}} + \sum_{i=1}^{n-1} I_{\Theta_{i}^{n-1}} - \sum_{i=1}^{n} I_{\Theta_{i}^{n-1}$$

The term  $I_{\Theta_i^{n+1}} + \sum I_{\Theta_i^{n-1}}$  is a sum of  $n I_{\Theta_i}$  with disjoint  $\Theta_i$ . By d hand side of the inequality is less than or equal to 0 and the left hand greater than or equal to 0, which is absurd.

If  $\forall i = 1...n + 1$ ,  $\exists j = 1...n - 1$ ,  $\Theta_i^{n+1} \cap \Theta_i^{n-1} \neq \emptyset$ . In that case  $[1, n] \Theta_i^n \cap \Theta_i^{n-1} \neq \emptyset$ , otherwise due to lemma 3.2 this would lead

If the condition of Lemma E.4 is not satisfied one can find a counte  $u_n^y < u_n^y - u_{n-1}^y$  and the equivalence of utility maximisation and optimised optimized by  $u_n^y = u_{n-1}^y$ . is not guaranteed. Finally, we have the desired result.

THEOREM E.5. Let us consider a dataset y and suppose that it arability at all orders,  $n = 1...n_{max}$  then there exists an increasing that (63) and (64) have the same solution, and a function  $\gamma'(x) > 0$ have the same solution.

PROOF. Under the hypothesis of separability, by lemma E.4, ( increasing, and by lemma E.3, we have the desired result.

#### APPENDIX F: OTHER DEFINITION OF MISSED DE

In this appendix, we show that the optimal procedure is similar in defined as in Hara et al. (2022b).

DEFINITION F.1 (Missed detections: other definition). If  $n \mod n$ in fact there are n' > n components truly present in the data, we could be a set of the data of the tions.

#### In that case the expected utility is

(38)

Let us consider  $\Theta_{n+1} \in T$ . The solution to  $(P_{n+1})$  can be written as

$$\arg \max_{\substack{\Theta_1 \in T \setminus \Theta_{n+1}, \dots, \Theta_n \in T \setminus \Theta_{n+1} \\ \forall i, j \in [\![1,n]\!], i \neq j, \Theta_i \cap \Theta_j = \emptyset}} I_{\Theta_{n+1}} + \sum_{i=1}^{\infty} I_{\Theta_i}.$$

Either  $\forall i \in [\![1,n]\!], \Theta_{n+1}^{n+1} \cap \Theta_i^n = \emptyset$  then thanks to (P1), for  $E = T^n$  and D  $T^n, \forall i, x_i \notin \Theta_{n+1}^{n+1}$ 

$$\arg \max_{\substack{\Theta_1 \in T \setminus \Theta_{n+1}, \dots, \Theta_n \in T \setminus \Theta_{n+1} \\ \forall i, j \in [\![1,n]\!], i \neq j, \Theta_i \cap \Theta_j = \emptyset}} \sum_{i=1} I_{\Theta_i} = (\Theta_i^n)_{i=1..n}$$

As a consequence

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 $I_{\Theta_{n+1}^{\star}}$  +

The

and mi

(57)

(60)

(61)

compo

(40) 
$$\arg \max_{\substack{\Theta_{n+1} \in T \setminus \bigcup_{i=1}^{n} \Theta_{i}^{n}}} \arg \max_{\substack{\Theta_{1} \in T \setminus \Theta_{n+1}, \dots, \Theta_{n} \in T \setminus \Theta_{n+1} \\ \forall i, j \in [\![1,n]\!], i \neq j, \Theta_{i} \cap \Theta_{j} = \emptyset}} I_{\Theta_{n+1}} + \sum_{i=1} I_{\Theta_{i}} = (\Theta_{1}^{n}, \Theta_{i}) = (\Theta_{1}^{n})$$

up to a permutation of the indices (see remark B.2). If  $\exists i \in [1, n+1], \forall j$  $\Theta_{i}^{n} = \emptyset$  then the same argument applies and the solution to  $(P_{n+1})$  is  $(\Theta_{1}^{n}, ..., \Theta_{n+1})$ Let

 $[\![1,n]\!], ($ CONTINUOUS MULTIPLE HYPOTHESIS TESTING FOR OPTIMAL cases a

Theorem E.1 assumes component separability (Definition 3.3) than necessary for some of the lemmas, and is not made by de throughout the appendix that the  $\Theta_i$ s are pairwise disjoint. T Eq. (4), we can write  $n - \sum_{i=1}^{n} j I_{A_i} = \sum_{i=1}^{n} \text{FIP}_{\Theta_i}$ . We consid and define

(41) 
$$u_n := \sum_{i=1}^n \operatorname{FIP}_{\Theta_i^n}$$

ent to

 $v_n := \sum (k-n)p(k \mid y) ; v'_n = \bar{n} - n - \bar{n}$ (42)

where  $\bar{n} = \sum_{k=1}^{n_{max}} kp(k \mid y)$ . The sequence  $u_n$  is the expected nu and  $v'_n$  are the expected number of missed detections for the mi F.1 and 2.3, respectively (see Appendix A for details). Note that but we chose not to make that dependence explicit to simplify no

LEMMA E.2.  $\forall y$  in the sample space the sequence (v  $(v_n)_{n=1..n_{max}}$  and  $(v'_n)_{n=1..n_{max}}$  are decreasing.

**PROOF.** Let us suppose that there exists n such that  $u_{n+1} < n$ 

$$n+1 - \sum_{i=1}^{n+1} I_{\Theta_i^{n+1}} < n - \sum_{i=1}^n I_{\Theta_i^n}$$

$$1 + \sum_{i=1}^{n} I_{\Theta_{i}^{n}} < \sum_{i=1}^{n+1} I_{\Theta_{i}^{n+1}}$$

by  $i_0$  an index such that  $i_0 = \arg \max_{i=1..n+1} I_{\Theta_i}^{n+1}$ 

$$n+1$$

$$+ \left[\sum_{i=1}^{n} -(n-i)\alpha I_{A_{i}^{n+1}} - n\alpha \left(1 - \sum_{i=1}^{n} I_{A_{i}^{n+1}}\right) - \beta\right] p(n+1 \mid y)$$

$$+ \left[\sum_{i=1}^{n} -(n-i)\alpha I_{A_{i}^{n}} - n\alpha \left(1 - \sum_{i=1}^{n} I_{A_{i}^{nmax}}\right) - (n_{max} - n)\beta\right] p(n_{max} \mid y)$$

$$+ \left[\sum_{i=1}^{n} -(n-i)\alpha I_{A_{i}^{n}} - n\alpha \left(1 - \sum_{i=1}^{n} I_{A_{i}^{nmax}}\right) - (n_{max} - n)\beta\right] p(n_{max} \mid y)$$

$$+ \left[\sum_{i=1}^{n} -(n-i)\alpha I_{A_{i}^{n}} - n\alpha \left(1 - \sum_{i=1}^{n} I_{A_{i}^{nmax}}\right) - (n_{max} - n)\beta\right] p(n_{max} \mid y)$$

$$E_{\theta,\eta}[U\{a,(\theta,\eta)\}] = -n\alpha + \alpha \sum_{i=1}^{n} iI_{A_i} - \beta \sum_{k=n+1}^{n} (k-n)p(k \mid y)$$

where E[FD] and E[MD] are the expected numbers of false detections and missed detections, an incre respectively.

$$E[FD] = n - \sum_{i=1}^{n} iI_{A_i}$$
$$E[MD] = \sum_{i=1}^{n_{max}} (k-n)p(k \mid y)$$

Assuming that  $\alpha \neq 0$  (or equivalently  $\alpha > 0$ , since  $\alpha$  is non negative), we can divide Eq. (59) by  $\alpha$ . Denoting by  $\gamma = \beta/\alpha$ , without loss of generality we can maximize

$$E_{\theta,\eta}\left[U\left\{a(\Theta_1,...,\Theta_n),(\theta,\eta)\right\}\right] = -n + \sum_{j=1}^n jI_{A_j} - \gamma \sum_{k=n+1}^{n_{max}} (k-n)p(k \mid y)$$

k=n+1

The expected utility is

(63)  

$$E = \left[ U \left( q(\Theta_{1} - \Theta_{2}) \left( \theta_{1} - \eta_{2} \right) \right] = -\eta_{1} + \sum_{i=1}^{n} i U_{i} \left( \Theta_{1} - \Theta_{2} \right) - q_{i} \sum_{i=1}^{n_{\max}} \left( h_{i} - \eta_{2} \right) r_{i}(h_{i}(h_{i}))$$



# An optimal exoplanet detection criterion

- Mathematical proof of optimality of a new detection criterion called « True inclusion probability » (TIP)
- Optimal in a general case

Hara et al. 2023, Annals of Applied Statistics (in revision) Hara, Unger, Delisle, Díaz, Ségransan 2022

Bayes factors and FAPs

• Optimal?

### New criterion demonstrably optimal

• Do not encode where the planet is

Encoded in new criterion

Are not defined on a very intuitive scale

→ New criterion is an actual probability



| 90% | 90% |
|-----|-----|
| 50% | 50% |







# Understanding stellar variability to correct it

Detailed physical models Analysis of solar data



Jour julien

### Numerous processes at Different timescales



Magnetic activity p.ex. Meunier+ 2010, 2012, 2019 Boisse+ 2012, Dumusque+ 2014 Haywood+ 2016, Al Moulla 2023 Granulation and super granulation Cegla+2013, 2019 Dravins+ 2021,

+ oscillations, winds, gravitational redshift...

#### **Data analysed with a statistical model** supposed Gaussian and stationary with qualitative parameters

Build the statistical model from a physical one







# Understanding stellar variability to correct it

Detailed physical models Analysis of solar data



Jour julien

### Numerous processes at Different timescales



Magnetic activity p.ex. Meunier+ 2010, 2012, 2019 Boisse+ 2012, Dumusque+ 2014 Haywood+ 2016, Al Moulla 2023 Granulation and super granulation Cegla+2013, 2019 Dravins+ 2021,

+ oscillations, winds, gravitational redshift...

#### **Building the statistical model**

#### Appendix A: Covariance and cumulants

point correlation function. For

Appendix A.1: Commission in the second

| In this ap                                  | ppendix,                                                                                                        | order 3 and 4 are zero. Our main re-                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             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| and autoc<br>and $z(t)$ ]                   | covariance<br>These exit                                                                                        | Theorem 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              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(A.22), we will u                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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| mula for                                    | cumulan                                                                                                         | Theorem 1.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             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         | $+\frac{1}{2}\cos\phi + \frac{1}{2}$                                                                | $-\cos 2\phi$                                                                                  | (B.37)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 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| manipulat                                   | tion of c                                                                                                       | $\kappa_n(y(t_1), y(t_2),, y(t_n)) =$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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| here give                                   | a calcul:                                                                                                       | $\iint g(t-t, x)g(t-t, x) = g(t-t, x) d(t)g(x)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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                 | $\mathbf{v} = \sum_{n=1}^{N} \mathbf{a}(\mathbf{T})$ | F(x) = f(x)A(x)F(x)I(x)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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| Let us                                      | s conside                                                                                                       | $\iint g(t-t_1, y)g(t-t_2, y)g(t-t_n, y)A(t)p(y)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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                 | $I_i = \sum_{k=1}^{n} g(I_i)$                        | $\Delta t_k, \ \Delta T(\mathbf{x}) = \Delta f(\mathbf{x}) A(\mathbf{x}) T(\mathbf{x}) t(\mathbf{x})$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | For a constant rotation rate of                                                                     | Moreover, the Zeeman                                                                           | broadening ef                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             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| interval [·                                 | -L/2, L                                                                                                         | The covariance is the 2-point correlation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              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| pearance                                    | follow 1                                                                                                        | Gaussian processes they are stationary, we                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             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                                                                                                                                                                                                                                                                                       | $(t_2) = are independe$                                                                                                                                  | nt, $\kappa(Y_1,, Y_i,, Y_i)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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Fin                                                            | ally, the effect                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| signal is r                                 | modellec                                                                                                        | $\kappa_n( t_1-t_2 )$ they are equivalently represented b                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              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                 | $t_1 - I/2$ and $t_n$                                | +1 $\phi$ and latitude $\delta$ ) on an in                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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| $L_{1}$                                     | n la companya da companya d | form of the kernel $\kappa(\tau)$ : their power spectru                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    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| $V^{L}(t) = \sum_{i}$                       | $\sum_{i} g(t-t_i)$                                                                                             | Similarly, under the hypothesis of stationarity quantity $\kappa$ (t t + $\tau_1$ , t + $\tau_2$ , does not dee                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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                                                                                                                      | where $c_0, c_1, c_2, s_1, s_2$ are coeffic                                                         | $\delta \text{CCF}(v) = \text{CCF}_s(v) -$                                                     | $CCF_l(v).$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               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| <i>κ</i> =                                  | =1                                                                                                              | ering the <i>n</i> point correlation function as a function function $n = 1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           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                 | Thanks to                                            | the radius. For the sake of sin                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | $\beta$ . As a concluding remark, let                                                               |                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| and we de                                   | enote by                                                                                                        | variables $\tau_1,, \tau_{n-1}$ , we can define the polys                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              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| spots such                                  | h that $t_{i}$                                                                                                  | dimensional Fourier transform of $\kappa_n(t, t + \tau_1,$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             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                 |                                                      | consider a direct frame x, y                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   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| ion of no                                   | aromotor                                                                                                        | The proof of Eq. (A.22) can be modified s                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 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                                                                                                    | ardly to $\kappa(Y_1,, Y_n) =$                                                                                                                           | $\sum \kappa(\kappa(Y_i:i\in B \mid$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    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                                           | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            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| of y which                                  | ch by det                                                                                                       | prove a more general result,                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           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                 | x(11,,1 <sub>n</sub>   11)                           | with z denote by $\phi$ , $\delta$ the st                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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| or y, with                                  | ch by dei                                                                                                       | $\kappa_n(x_1(x_1), x_2(t_2),, x_n(t_n)) =$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            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| $Cov(y(t_a))$                               | $(y(t_b)) =$                                                                                                    | $\iint f(x, y, y) = f(x, y) =$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         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| we first c                                  | compute                                                                                                         | $\iint f_1(t-t_1,\gamma)f_2(t-t_2,\gamma)\dots 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| $p(\gamma, t) dt d\gamma$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | (A.22) means that B                                                                                                                                      | runs through the                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          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- \frac{c_l o_l}{c_l o_l} \exp \left[\frac{1}{c_l o_l} + \frac{c_l o_l}{c_l o_l}\right]$ | $-\frac{1}{2}\frac{cr_{l}}{r^{2}+r^{2}}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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| product e                                   | expectanc                                                                                                       | where $r_{1}(t) = v(t)$ or $r_{2}(t) = z(t)$ . If $r_{2}(t) = v(t)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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                                                                      | $= q(t = k(Y_i : i \in X_i))$                                                                                                                            | $B \mid X$ is the                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Appendix B.2. CCF-related cha                                                                       | $\sqrt{\sigma^2 + \sigma_l^2}$                                                                 | $2\sigma^2 + \sigma_l^2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| ation for                                   | y <sup>2</sup> and I                                                                                            | and if $x_i(t) = z(t)$ , then $f_i(t) = h(t)$ . This exp                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 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| The a                                       | iverage v                                                                                                       | eq (A.18). This expression has the advantage                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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                                                                                                          | d if the compute $\kappa(Y)$                                                                                                                             | $(Y_2, Y_2)$ . The parti                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | in Theorem A.                                        | $\frac{2.2}{2.2}$ The local frame $(u, v, w)$ at                                                                                                                                                                                                                                                                                                                                         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| $\mathbb{E}\left\{y_{L}(t)\right\} =$       | $-e^{-I_{\lambda}^{L}}\sum^{+\infty}$                                                                           | process is non stationary, and can explore no                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          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                                                                                                                                                                                                                                                                                       | depen- are $\{\{1\}, \{2\}, \{2\}, \{2\}, \{2\}, \{2\}, \{2\}, \{2\}, \{2$                                                                               | $\{3\}\}, \{\{1\}, \{2, 3\}\}, \{\{2\}\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\nu(G(T_1 - t_1))$                                  | The local Halle $(u, v, w)$ at                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | derive the expressions of PV as y                                                                   | $-a_s \frac{1}{\sqrt{\sigma^2 + \sigma^2}} \exp\left(\frac{1}{\sigma^2 + \sigma^2}\right)$     | $-\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_z^2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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| B(y (i)) -                                  | - C ^ Z                                                                                                         | dencies across channel. However, it is unlike                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    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                                                                                             | nese as- In the partitio                                                                                                                                 | n {{1}, {2}, {3}} we l                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         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In the part                                                                                                                                      | ition {{1}, {2, 3}} we                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                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| And we h                                    | nave                                                                                                            | rate, and the distribution followed by $\gamma(t_k)$ d                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 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                                                                                                                      | When extracting radial-veloc                                                                        | with $\delta v_l = v_l - v_0$ , $\delta v_s$                                                   | $= v_s - v_0$ . At le                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|                                             | luve                                                                                                            | time, $\gamma(t_k) \sim p(\gamma)$ , the stochastic process $y(t_k)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   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We $\kappa(Y_1, Y_2, Y_3) =$                                                                                                                        | $\kappa(\kappa(Y_1, Y_2, Y_3 \mid X))$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             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| $\sum g$                                    | $g(t-t_k, \gamma)$                                                                                              | can write Eq. $(A.22)$ as a function of time diff                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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| $JJ \underset{k=1}{\underline{\checkmark}}$ |                                                                                                                 | with the change of variable $t \leftarrow t - t_1$ ,                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   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                 | At each $i$ th                                       | star with respect to the plane                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $m_{\rm CCF}(v;\eta) = a \left[ 1 - c \exp \left[ -\frac{1}{2} \right] \right]$                     | $b_0 \approx a_1 \left[1 - \frac{c_s \sigma_s}{c_s \sigma_s}\right]$                           | $-   - a_1   1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           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| <u>n</u> (                                  | $\int a(t-t)$                                                                                                   | $\kappa_{n}(\tau_{1},,\tau_{n-1};n) = \lambda \iint g(t,\gamma)g(t+\tau_{1},\gamma)g(t+\tau_{n-1};n) = \lambda \iint g(t,\gamma)g(t+\tau_{n-1};\gamma)g(t+\tau_{n-1};n) = \lambda \iint g(t,\gamma)g(t+\tau_{n-1};\gamma)g(t+\tau_{n-1};\gamma)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       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                                                                                                                                                                                                                                                                                       | $n$ )dtd $\gamma$ .                                                                                                                                      | $\kappa(\kappa(I_1, I_3 \mid X), \kappa(I_2 \mid X))$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                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| $I_{\lambda}^{L}$ JJ                        | 8(1 -1                                                                                                          | JJ school by sc                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | <i>n</i> -1, <i>1</i> , 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | (A 23)                                                                                                                                                   | $\kappa(\kappa(I_2, I_3 \mid A), \kappa(I_1 \mid X))$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $N) = \kappa(G(T_1$                                  | Sudden appearing and exp.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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\rho e$                                                            | $\exp(- \tau /\rho_{-}) \qquad \qquad$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|                                             | $+\infty$                                                                                                       | In that case, the Fourier transform of $\kappa_n$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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                                                                                                                                                                                                                                                                                       | tion of where the qua                                                                                                                                    | intities $\kappa(Y_i : i \in I)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\kappa(Y_1,, Y_n   .$                               | Asymmetric exp. (dis)appea                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | along $\mathbf{I}_{t<0} \exp(t/p_{-}) + \mathbf{I}_{t>0} \exp(-p_{-})$                              | $(1,  \tau ) = \frac{\rho_+ - \rho}{\rho_+ - \rho}$                                            | Su                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| $\mathbb{E}\{y^L(t)\} =$                    | $= e^{-I_{\lambda}^{L}} \sum$                                                                                   | $\tau_1,, \tau_{n-1}$ is called the polyspectrum of order<br>the integrals on $\gamma$ and $\tau_1,, \tau_{n-1}$ can be inver                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | r n. 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| $\mathbb{E}\{y^{n} t_{a}\}$                 | $\hat{I}(v,\phi_0,\gamma)$                                                                                      | $) = \sum \hat{W}(\nu - k\omega, \gamma)i_{k}(\gamma) \exp(ik\phi_{0}),$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               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                 | e of $\phi$ in the ra                                | where $\mu_i(\eta) = \mathbb{E}_{\gamma}(l_0(\gamma)), \ \mu_j(\eta)$<br>Eq. (C.31) is exactly the same                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\beta = \mathbb{E}_{\gamma}(f_0(\gamma))$ . The first term in<br>as in the case of a constant rate | = N(N-1) $P(t)$                                                                                | $P(t+\tau)w(t)w(t)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| $\sum_{\substack{i=1,k\neq i}}$             | where $i_k$ i                                                                                                   | is the Fourier coefficient                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             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| <i>n</i>                                    | $i_k(\gamma) = \frac{1}{2}$                                                                                     | $\frac{1}{\pi} \int_{0}^{1} i(\phi, \gamma) \exp(-ik\phi) d\phi.$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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|                                             | The same                                                                                                        | e is true for J and the Fourier transform of $k^{2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   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                                                                                                                      | nted in Section 5.11 Let us recall                                                                  | now marginalizing on                                                                           | iv we obtain                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              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|                                             | $\hat{k}^{I,J}(\nu,\phi_0,$                                                                                     | $(\gamma, \gamma) = \sum \hat{W}(\nu - k\omega, \gamma)\hat{W}(\nu - l\omega, \gamma)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 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                                                                                                                      | e that granule packets appear at                                                                    | $k = (\tau) = \lambda T \int_{-\infty}^{\infty} P(t)$                                          | $P(t \perp \sigma) \dots (t \rightarrow t)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               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                                                                                                                      | ess $\lambda_0$ . If a granule packet appears                                                       | $\kappa_{gran}(t) = \Lambda I \int_{-\infty} P(t) dt$                                          | $t(t+\tau)w_2(t,\tau)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
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(2022), this r                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           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|                                             | We now                                                                                                          | integrate this over the longitude at maximum                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           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                 | ected by the sa                                      | $[t_0, t_0 + T]$ , As a result. N follo                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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| 1                                           | $\int_0^{\infty} \exp($                                                                                         | $(1(l-k)\phi_0)\frac{1}{2\pi}=\delta_{k,l},$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           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| 1                                           | $\int_{1}^{2\pi} \hat{I} J J$                                                                                   | $(u, \phi, u)^{\mathrm{d}\phi_0} = \sum  \hat{u} ^2 (u, -k_0, u)^{\mathrm{d}\phi_0}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   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|                                             | $\int_0^\infty k^{-\infty}$                                                                                     | $(v, \varphi_0, \gamma) \overline{2\pi} = \sum_{k \in \mathcal{T}}  w  (v - \kappa \omega, \gamma) l_k(\gamma) J_k(\gamma).$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           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                 | $(i_{I_a,k}\exp(\mathrm{i}k\omega t))$               | (6), and we replace the impuls                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 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|                                             | $k^{y,z}(\tau, n)$                                                                                              | $= \lambda \int k^{W}(\tau, \gamma) k^{i,j}(\tau, \gamma) p(\gamma   n) d\gamma$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       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To simplify notations, we write                                                                   | $f_{gran}(t, \gamma) \ll \Lambda I \int_{-\infty}^{\infty} W$                                  | $(i) m (i + \tau) u +$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
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The covar rank $r = 2k + 1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | ance of Eq. 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# Phase shifts and faculae/spot ratios





## **Statistical** Doppler imaging



Finding the solar inclination from RVs (HARPS-N) and photometry (SORCE)

To be refined



### Statistical model with physical parameters

(In review)





# **Application: super-Terres dans la zone habitable**



Large program HARPS (PI) 340h to confirm these detections (with X. Dumusque, M. Crétignier, N. Unger) Oct. 2022- Oct. 2023

stars

Planètes connues à mains de 15 pc







# Conclusion



Radial velocity measurements would greatly improve the scientific yield of 2050 flagship missions

Detecting Earth like planets with radial velocities is a very difficult problem but progress is made

Results of the large program end of next year







#### Solar system



# Generate realisations of the FENRIR model

(1) Draw times at which features appear according to a variable rate  $\lambda(t)$ 





(2) If a feature appears at time t draw its parameters  $\gamma$  from distribution  $p(\gamma \mid \eta, t)$ 



























# From the physical model, compute the covariance of the likelihood function

$$\iint g(t-t_1,\gamma)h(t-t_1',\gamma)\lambda(t$$

$$\iint g(t - t_1, \gamma)g(t - t_2, \gamma)\lambda(t)p(\gamma \mid t, \eta)dtd\gamma$$

$$\iint h(t - t'_1, \gamma) h(t - t'_2, \gamma) \lambda(t) p(\gamma \mid t, \eta) dt d\gamma$$

$$\iint l(t - t'_1, \gamma)h(t - t'_2, \gamma)\lambda(t)p(\gamma \mid t, \eta)dtd\gamma$$















**Exoplanet** observations

### Characterization of exoplanetary atmospheres

### Detecting life outside of Earth







### Characterizing Earth-analogs Detecting life outside of Earth

#### Spectro-imaging: LIFE (ESA), LUVEX (NASA), JWST), PCS@ELT



#### Thermal emission: LIFE, ESA

**Reflected light:** HWO, NASA





# **Objectives**



# **Gives the ratio** Time **Of planetary and Stellar radii Requires very** Specific system geometry





# **Radial velocites are key**



Morgan et al. 2021



Mass is essential to interpret spectroscopic observations of atmospheres (Batalha et al. 2017, 2019)

The yield of atmospheric characterization is greatly improved if Earth like planets are detected in advance, require targets  $\leq$  20 parsec

- + Results obtained faster, more time for characterisation
- + Mitigates the risk of the missions
- + True for other kinds of planets

